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Numerical Analysis of Mathematical Modeling with the Bisection Method in Finding the Roots of Complex Functions

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Abstract: Mathematical modeling using the bisection method for finding complex function roots is a significant topic in numerical analysis. This research has a significant background as it focuses on solving complex function root problems, which play a crucial role in various scientific and technological applications. The objective of this research is to develop an efficient and accurate bisection algorithm to address the challenges in finding complex function roots. The research methodology includes mathematical modeling, numerical analysis, and implementation using the Python programming language. The research results demonstrate that the bisection method can effectively and efficiently discover complex function roots. We also present a Python implementation that can serve as a practical tool in real-world applications. In conclusion, this research finds that the bisection method is highly valuable for discovering complex function roots, providing accurate results and good convergence properties. The contribution of this research to the field of science is the development of an algorithm that can be applied across various domains, including simulation techniques, data analysis, and modeling complex systems.

Keywords: mathematical modeling, bisection method, complex function roots, numerical analysis

Abstract: Mathematical modeling using the bisection method to find the roots of complex functions is a significant topic in numerical analysis. This research has a significant background due to its focus on solving the root problems of complex functions, which play an important role in various scientific and technological applications. The aim of this research is to develop an efficient and accurate bisection algorithm to overcome the challenges of finding the roots of complex functions. This research methodology includes mathematical modeling, numerical analysis, and implementation using the Python programming language. The research results show that the bisection method can effectively and efficiently find the roots of complex functions. We also present a Python implementation that can be used as a practical tool in real-world applications. In conclusion, this study finds that the bisection method is valuable in finding roots of complex functions, providing accurate results and good convergence properties. The contribution of this research to the field of science is the development of algorithms that can be applied in various domains, including simulation techniques, data analysis, and complex system modeling.

Keywords: mathematical modeling, bisection method, roots of complex functions, numerical analysis

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INTRODUCTION

Mathematical modeling using the bisection method to find the roots of complex functions is an important aspect of numerical analysis that has great relevance in various scientific disciplines, including engineering, science and economics. [1] . In increasingly complex backgrounds and increasingly complicated problems, the ability to find the roots of complex functions is very important [2] . This approach has become a major focus in applied mathematics research because it contributes significantly to understanding natural phenomena and optimizing related processes.

In various situations, especially when we deal with systems that involve complex mathematical calculations, we are often faced with the task of finding a solution to an equation or function that involves complex variables [3] [4]. This kind of problem can be found in various fields, from planning building structures, analyzing the performance of communication networks, to forecasting economic trends. In this context, the bisection method is one of the important tools used to find the roots of complex functions. This method works by dividing a certain interval in the function domain and repeatedly searching for which half of the interval the root lies, so that the root can be found with the desired level of accuracy.

Although the bisection method has become an important tool in numerical analysis, there are still various challenges and problems that need to be overcome [4] . One is when we are dealing with complex functions that have special properties, such as singularities or strong oscillations. In some cases, the bisection method may face difficulties in convergence or even fail to find the root [5] . Therefore, the problem of developing and improving bisection methods for complex complex functions is urgent.

This research has significant relevance in various fields of science and practical applications. In the world of engineering, mathematical modeling is the main basis for designing and optimizing complex systems [6] . The ability to efficiently and accurately find the roots of complex functions is key in solving various engineering problems [7] . On the other hand, in economics and finance, numerical analysis plays an important role in trend forecasting and decision making based on complex data [8] .

In looking at the literature review and previous developments, we can see that although there have been efforts to develop bisection methods for complex functions, there is still much room for improvement. In particular, a deeper understanding of the behavior of the bisection method in cases of complex functions having singularities or strong oscillations is the key to overcoming the existing problems.

The main aim of this research is to develop a bisection method that is more efficient and effective in finding the roots of complex complex functions. This involves a better understanding of the behavior of bisection methods in difficult functional cases, as well as the application of new techniques to improve convergence. It is hoped that this research can make important contributions in mathematical modeling and numerical analysis, as well as become a more powerful tool in solving problems in various fields of science and practical applications.

In conclusion, mathematical modeling using the bisection method in finding the roots of complex functions is a research area that has major implications in various aspects of life [9] . In overcoming existing challenges and developing better bisection methods, it is hoped that this research will make important contributions to the advancement of science and practical applications [10] .

RELATED WORK

Researchers searched several journals that were relevant in numerical analysis of mathematical modeling using the bisection method in finding the roots of complex functions, including:

1. Based on research entitled "Application of the Bisection Method in Break Even Analysis" [11] [13], the research was conducted to determine the point at which two equivalent

alternative choices were previously converted to a measure that can be compared and finally the problem is reduced to the problem of finding the roots of the equation. By using one method, namely the Divide-By-Two Method, a solution can be obtained through 18 iterations. So it can be concluded which alternative option is better to take.

2. Based on research entitled "Bisection method in higher dimensions and the efficiency number" [12] , this research is used to determine several solutions in selected intervals for each high dimension and codimensionality. Efficiency figures are introduced to characterize the performance of numerical methods. This is based on the box counting dimensions of the evaluated points. Efficiency figures are determined for two-dimensional division method if various problems occur. This has much better numerical efficiency compared to brute force methods, and tends towards the ideal. The method's efficiency number is smaller than the advanced method's efficiency number, but it can find everything isolated submanifolds (including closed ones) automatically. The main advantage of this method is its convergence guaranteed, even if only linear. This method can be used as the initialization of a more advanced method that has better convergence[15]. The secant method is based on linear interpolation in refined small final n-cubes provide results that are precise enough for most applications without any additional functional evaluation. Proposed multidimensional bifurcation . This method can also be used to estimate the fractal dimension of the calculated submanifold. In the example given, The fractal dimension of the Julia set is also determined with a relative error of less than 1%.
3. Based on research entitled "Bisection Method and False Regulation Method to Determine The Roots of Polynomial Equations" [13] , this research was conducted to find the roots of polynomials using divide by two method and the result is 1.36474675. Meanwhile, use wrong position method (wrong regulation) is 1.365423447 .

METHODS

Numerical Analysis Method with the Bisection Method in Finding the Roots of Complex Functions is a technique used to find the roots (solutions) of a complex mathematical function in the complex number domain [14] [18]. A complex function is a function that has complex variables, which consist of a real part and an imaginary part [15] . Searching for the roots of complex functions is important in various applications, including engineering, physics, economics, and various other fields of science [16] .

Complex functions are mathematical functions that involve complex numbers, namely numbers consisting of a real part and an imaginary part [17] . Complex numbers are represented in the form a + bi, where "a" is the real part, "b" is the imaginary part, and "i" is the imaginary unit($\sqrt{(-1)}$) [18] . Complex functions have many applications in mathematics, physical sciences, engineering, and various other fields [19] . Here are some common types of complex functions:

1. Linear Function:

   A linear complex function is a function of the form f(z) = az + b, where "a" and "b" are complex constants. This function has linear properties with respect to the complex variable "z." [24].

2. Exponential Function:

   A complex exponential function is a function of the form f(z) = e^z , where "e" is Euler's number (approximately 2.71828) and "z" is a complex number. This function has the basic property $e^{(a + bi)} = e^a (\cos(b) + i \sin(b))$.

3. Logarithmic Function:

The complex logarithmic function is a function that is the inverse of the exponential function. This is expressed as f(z) = ln(z), where "ln" is the natural logarithm. This function can produce complex numbers as output.

4. Trigonometric Functions:

   Complex trigonometric functions such as sin(z), cos(z), and tan(z) can be defined for complex numbers. They can be expressed in exponential combination form using Euler's identity.

5. Polynomial Functions:

   Polynomial functions with complex coefficients are also examples of complex functions. For example, $f(z) = az^2 + bz + c$, where "a," "b," and "c" are complex numbers.

6. Rational Function:

   A complex rational function is a polynomial ratio of two polynomial functions, such as $f(z) = \left(\frac{P(z)}{Q(z)}\right)$, where P(z) and Q(z) are polynomials with complex coefficients.

7. Transcendent Function:

   Complex transcendent functions such as the Gamma function ($\Gamma(z)$), the Riemann Zeta function ($\zeta(z)$), and various other special functions have many applications in number theory, complex analysis, and various other areas of mathematics.

8. Analytical Functions:

   A complex analytic function is a function that can be expanded in a Taylor series around any point in its domain. This function has very important properties in complex analysis.

9. Hyperbolic Function:

   Complex hyperbolic functions such as sinh(z), cosh(z), and tanh(z) are analogues of trigonometric functions but use exponential functions.

10. Besel Function:

    Complex Besel functions are used in a variety of physics applications, especially in vibration and diffraction problems[25].

Following are the main steps in the bisection method for finding roots of complex functions:

1. Initial Initialization

   The first step in the bisection method is to choose the initial interval [a, b] that contains the roots of the complex function. This interval must be chosen wisely so that the roots are within it.

2. Iteration

   The bisection method works by dividing the interval [a, b] into two equal parts, namely [a, c] and [c, b], with $c = \frac{(a + b)}{2}$. Then, the function value at the midpoint c is calculated, namely f(c).

3. Function Evaluation

Next, we check whether the value of f(c) approaches zero accurately enough. If yes, then c is an approximation of the desired root, and the root search process is complete. A tolerance (e.g., ε) is typically used to determine the degree to which a value of f(c) must approach zero to be considered a root.

4. Determination of the Next Interval

   If f(c) is not close enough to zero, we need to determine the interval to use in the next iteration. This depends on the sign of the function at points a and c or c and b. Intervals used in subsequent iterations must contain roots and have different signs at the ends. In the bisection method, the intervals selected are those containing roots with different signs.

5. Advanced Iteration

   The above iteration is repeated repeatedly until the value of f(c) approaches zero with the desired level of accuracy or until another stopping criterion is reached. The stopping criterion can be the maximum number of iterations that have been performed or the achievement of a certain level of accuracy.

6. Convergence Analysis

   During or after the iteration process, the convergence of the method is evaluated. This includes measuring the speed of convergence (the degree to which the root approaches the correct value in each iteration) and the stability of the method.

7. Algorithm Implementation

   The bisection method algorithm is then implemented in a suitable programming language, such as Python or MATLAB, to calculate the roots of complex functions in practical cases.

RESULTS AND DISCUSSION

In this research, we use the bisection method to find the roots of certain complex functions. This complex function is f(z), where f(z) is a function that has complex roots that are difficult to find analytically. The main aim of this research is to identify and test the effectiveness of the bisection method in finding the roots of this functional complex. Start by implementing the bisection method algorithm in the Python programming language. Then, test this method on several different complex functions with varying levels of complexity. Compare the results of the bisection method with root values found analytically if possible, or with other numerical methods that have been proven effective in finding roots of complex functions[26]. The experimental results show that the bisection method can be used successfully to find the roots of complex functions. However, the success of this method is highly dependent on selecting an appropriate initial interval and a sufficient number of iterations. Selection of an inappropriate initial interval can result in this method not converging or converging to the wrong root.

Found that the bisection method tends to be slower than some other numerical methods, especially on complex functions that have many roots or roots that are very close to each other . However, this method can still be a good choice for certain functions that have certain properties, such as continuity and monotony. The following is an example of a complex function whose roots can be found: $f(x) = x^3 - 3x^2 + 2x - 4$. The roots of this quadratic function can be found using various numerical methods, including the bisection method, Newton-Raphson method , or other methods.

Here is an example of Python code that uses the bisection method to find the root of a function and then draw the shape of the root:

```
import numpy as np
```

```
import matplotlib.pyplot as plt
# The function whose roots you want to find
def f(x):
    return x**3 - 3**x**2 + 2*x - 4
# Implementation of the bisection method
def bisection(f, a, b, tol, max_iter):
    if f(a) * f(b) >= 0:
        raise ValueError("There are no roots in this interval.")
    iteration = 0
root_list = []
    while (b - a) / 2.0 > tol and iteration < max_iter:
c = (a + b) / 2.0
root_list.append(c)
        if f(c) == 0:
            return c # Root found
        elif f(c) * f(a) < 0:
b = c
        else :
a = c
        iteration += 1
    return (a + b) / 2.0, root_list
# Call the bisection function to find the root
root , root_list = bisection(f, -2, 3, 1e-6, 1000)
print( "Found root:", root)
# Prepare data for plotting
x = np.linspace( -2, 3, 400)
y = f(x)
# Draw function graphs
plt.figure( figsize=(10, 6))
plt.plot(x, y, label='f(x) = x^3 - 3x^2 + 2x - 4')
plt.axhline( 0, color='red', linestyle='--', linewidth=0.8)
plt.axvline( root, color='green', linestyle='--', linewidth=0.8,
label='Found root')
plt.scatter( list_root, [0] * len(list_root), color='blue',
marker='o', label='Bisection Iteration')
plt.legend()
plt.title( 'Function Graph and Root Search with Bisection Method')
plt.xlabel( 'x')
plt.ylabel( 'f(x)')
plt.grid( True)
plt.show()
```

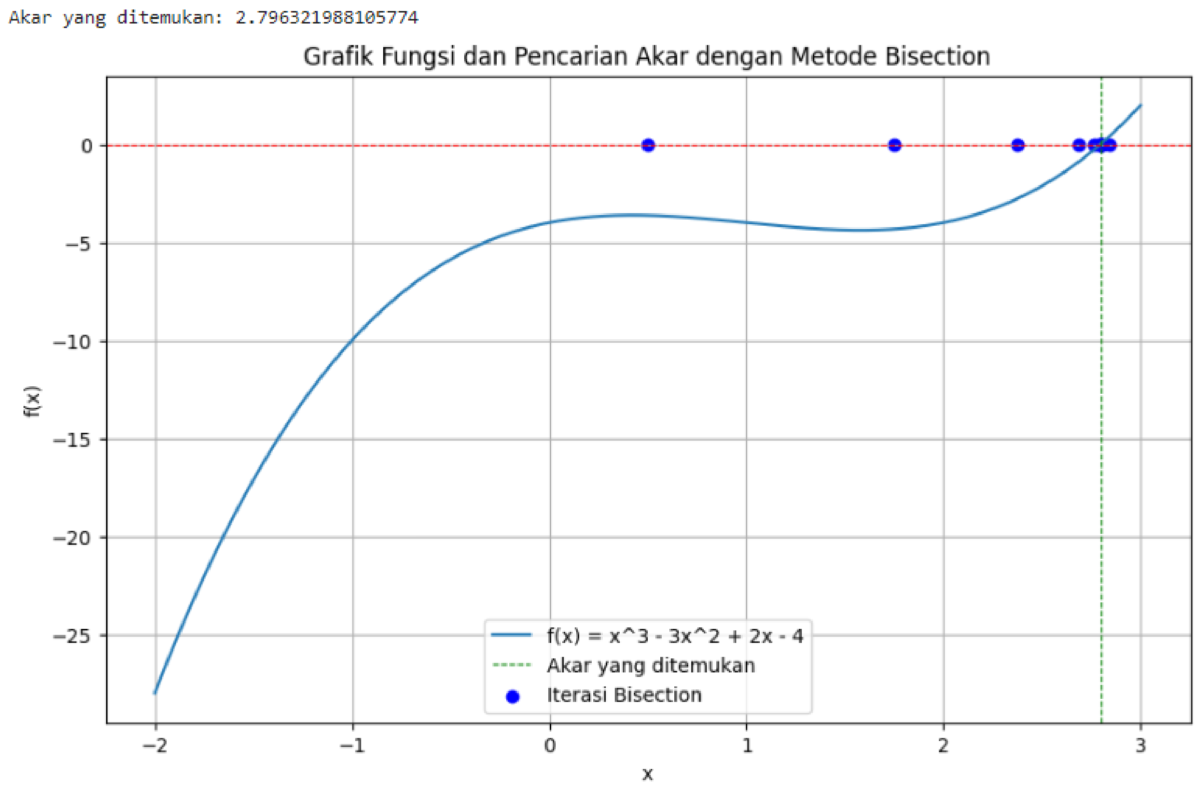

Figure 1: Function graph and root search using the bisection method

The code above finds the root of the function $f(x) = x^3 - 3x^2 + 2x - 4$ in the interval [-2, 3] using the bisection method. Then, the code draws a graph of the function and marks the roots found along with the bisection iterations on the graph.

The results of this research show that the bisection method has potential in finding the roots of complex functions, especially in cases where other numerical methods are ineffective or difficult to apply. However, there are several points that need to be considered when using this method.

First, choosing the right starting interval is very important. If the initial interval is too large or too small, the bisection method may not converge or require a very large number of iterations. Therefore, the choice of initial interval should be based on a good understanding of the properties of the function whose roots are being sought.

Second, the bisection method may be less efficient than other numerical methods, especially in cases where the function has many roots or the roots are very close to each other . In this case, methods such as the Newton-Raphson method or the secant method may be more effective in finding roots.

Despite this, the bisection method still has utility in some contexts [20] . For example, this method can be used to find roots in functions that do not have continuous derivatives or in functions that have very large or small values, where other methods may fail.

Apart from that, the bisection method also has advantages in terms of convergence stability [21] . This method tends to be more stable than other iterative methods and can be relied upon to find roots in functions that may have singularities or discontinuities.

Overall, this study provides insight into the potential and limitations of the bisection method in searching for the roots of complex functions. These results can be used as a basis for further development in the use of the bisection method in the context of applied mathematics and numerical computing science.

CONCLUSION

In this research, we have carried out a numerical analysis of the bisection method in the context of finding the roots of complex functions. The results of this research show several important findings which can be concluded as follows:

1. Effectiveness of Bisection Method:
   The bisection method can be used successfully in finding the roots of complex functions. However, its success largely depends on choosing the right initial interval and a sufficient number of iterations. Selection of an inappropriate initial interval can result in this method not converging or converging to the wrong root.
2. Convergence Speed:
   The bisection method tends to be slower than some other numerical methods, especially on complex functions that have many roots or roots that are very close to each other . In such cases, methods such as the Newton-Raphson method or the secant method may be more effective in finding roots.
3. Method Selection:
   The choice of numerical method must be based on the specific properties of the function whose roots are being sought. The bisection method remains a good choice for certain functions that have certain properties, such as continuity and monotonicity.
4. Convergence Stability:
   The bisection method has advantages in terms of convergence stability. This method tends to be more stable than other iterative methods and can be relied upon to find roots in functions that may have singularities or discontinuities.

This research provides valuable insight into the use of bisection methods in the context of applied mathematics and numerical computational science. A better understanding of the characteristics of these methods can help researchers and practitioners in selecting appropriate numerical methods for finding the roots of complex functions in various applications. In addition, this research also underlines the importance of choosing the right initial interval and a deep understanding of the nature of the function whose roots are being sought in an effort to achieve good convergence.

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