

Investment Optimization with Nonlinear Equation Solving

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Abstract: Investment optimization is one of the important topics in the world of finance that aims to maximize profits with minimal risk. Mathematical approaches, particularly through the solution of nonlinear equations, have become an effective method of aiding investment decision-making. This article discusses the development of an investment optimization model that uses nonlinear equation solving techniques to determine optimal asset allocation. In this study, a nonlinear equation is used to describe the relationship between various investment variables, such as profit level, risk, and asset allocation. Using this approach, investors can find optimal solutions that meet their investment goals, whether in conservative, moderate, or aggressive scenarios. The methodology used involves historical data analysis, mathematical model formulation, and the application of numerical algorithms to solve the nonlinear equations. The results show that the solution of nonlinear equations is able to provide a more precise solution than traditional methods, such as linear programming or simple heuristic. This approach not only improves accuracy in determining the optimal portfolio, but also provides flexibility in dealing with dynamic market conditions. The proposed model allows sensitivity analysis to variable changes, allowing investors to make more informative and adaptive decisions. Investment optimization with the solution of nonlinear equations is a significant innovation in the field of finance, which not only supports investment efficiency but also opens up opportunities for the development of more complex investment models. This article is expected to be a reference for academics and practitioners in applying a mathematical approach for optimal portfolio management.

Keywords: Investment optimization; Nonlinear equations; Optimal asset allocation; Financial decision-making; Investment sensitivity analysis; Optimal portfolio.

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INTRODUCTION

Investment has become an important element in financial management, both for individuals and companies [1]. With the increasing complexity of financial markets, the need for an effective approach to optimize asset allocation is becoming more and more urgent. One of the main challenges in investing is how to maximize profits while maintaining an acceptable level of risk. In this context, mathematical approaches play a key role in supporting informative and data-driven decision-making. One of the prominent methods is optimization by using the solution of nonlinear equations [2][3].

Nonlinear equations are tools that are able to describe the complex relationships between various variables in investments, such as the rate of return, risk, and asset allocation limits [4][5]. Unlike linear approaches, nonlinear models allow for more realistic analysis of dynamic market conditions and uncertainty [6]. This model provides flexibility in describing the nonlinear relationships between variables, which often reflect the irregular, unpredictable characteristics of financial markets. This makes the nonlinear equation a promising approach to solving the problem of investment optimization.

Various previous studies have highlighted the importance of using mathematical models in investing, but most are still limited to linear approaches[7]. This approach often ignores the complexity of the relationships between financial variables, resulting in less accurate solutions. In the real world, the relationship between risk and return is often not linear. For example, an increase in risk is not always directly proportional to an increase in returns, depending on factors such as market liquidity, economic uncertainty, and investor behavior. Therefore, a nonlinear approach offers a more in-depth and realistic solution to deal with the problem. The use of nonlinear equations in investment optimization not only provides a more precise solution, but also allows for a more comprehensive analysis of various investment scenarios [8]. With this approach, investors can evaluate the impact of variable changes on their portfolios, such as changes in interest rates, market volatility, or government policies. This model allows for the development of adaptive investment strategies, which are crucial in the face of ever-changing market conditions.

The main purpose of this article is to explore the application of nonlinear equation solutions in investment optimization. This article not only discusses the theoretical aspects of this approach, but also presents practical applications in portfolio management. In this analysis, various techniques for solving nonlinear equations, such as iterative methods, Newton-Raphson methods, and other numerical algorithms, will be discussed in detail [9]. Emphasis is also placed on the importance of model validation through historical data analysis and simulation of market scenarios. Furthermore, this article will discuss the challenges that may arise in the application of a nonlinear approach, such as the need for higher computing and difficulties in calibrating the model. With technological advancements and the availability of increasingly abundant data, these challenges can be overcome [10]. In the era of big data and artificial intelligence, the incorporation of nonlinear models with advanced technologies such as machine learning can open up new opportunities in investment management[11].

This approach is not only relevant for institutional investors, but it can also be applied by individual investors who want to maximize their portfolio returns. With the help of the software and analytical tools available today, even investors with limited mathematical understanding can leverage nonlinear models to support their decision-making [12]. This shows the potential for greater inclusivity in the application of modern financial technology. Investment optimization with the solution of nonlinear equations offers an innovative and relevant approach to facing the increasingly complex challenges of financial markets. This approach not only improves accuracy in decision-making, but also provides the flexibility needed to deal with uncertainty. This article is expected to provide new insights for academics, practitioners, and investors in applying mathematical methods to support investment success.

All discussions in this article are compiled to provide a thorough and in-depth understanding of this topic, starting from the theoretical foundation, methodology, to its practical application in the world of finance. This article is expected to be a significant contribution to the financial literature, especially in the field of mathematical-based investment optimization. The approach used in this article also highlights the importance of cross-disciplinary collaboration. The use of nonlinear equations in investment optimization is not only a relevant topic in applied mathematics but also includes elements of finance, economics, and information technology [13]. The role of data analysis, algorithm development, and understanding of market dynamics makes this topic highly interdisciplinary. In an increasingly digitally connected world, this kind of approach is becoming more accessible and

implemented by a wide range of stakeholders, from financial institutions to individuals involved in technology-based investments. In the academic context, this research is expected to contribute to the development of modern optimization theory. Many studies that have been conducted before have focused on simple linear optimization models, while studies on nonlinear optimization are still relatively limited, especially in the context of investment [14]. This article attempts to fill the gap by providing a strong theoretical foundation and relevant real case studies. The combination of theoretical and practical approaches is expected to motivate more further research in this area.

In addition to academic relevance, the practical benefits of this research are significant. Investors face great challenges in managing their portfolios amid market uncertainty. Changing global economic conditions, political turmoil, and rapid technological innovation can affect investment returns in unexpected ways. In this situation, having an analytics tool that can provide data-driven insights is a huge advantage [15]. By adopting the solution of nonlinear equations, investors can be better prepared for various scenarios, reduce the risk of loss, and maximize potential profits. Not only that, but the practical application of this approach also opens up opportunities to overcome challenges in sustainable investment management. With the growing awareness of the importance of green or environment-based investments, the nonlinear model can be used to optimize portfolios that include sustainable assets, such as renewable energy or green projects [16]. This approach allows investors to consider sustainability factors without sacrificing their financial goals.

It is important to note that the successful implementation of this method depends on a good understanding of the model and parameters used [17]. This article pays special attention to how to build an accurate model, select relevant parameters, and validate the results to ensure the reliability and credibility of the model. By using representative historical data and simulation techniques, investors can reduce the risk of bias in their decision-making. In closing, the development of an investment optimization model based on the solution of nonlinear equations is a step forward in the use of technology and mathematics for financial management. This approach provides a more comprehensive, adaptive, and realistic solution than traditional methods. With continued advances in computing technology and data availability, the potential for the application of this method will be even wider in the future. This article not only aims to provide theoretical insights but also serves as a practical guide for those who want to apply this method in a real-world context. This is expected to encourage wider adoption of a nonlinear equation-based approach in investment optimization, both in academia and industry.

RELATED WORK

The use of mathematical approaches to investment optimization has been the subject of extensive research in recent decades. Traditional methods such as linear optimization and Markowitz's model have become the basis of portfolio theory. Developed a modern portfolio theory based on a linear approach to minimize risk while maximizing portfolio returns [18]. This approach has limitations, especially in capturing complex market dynamics and nonlinear relationships between financial variables. The researcher tried to overcome the weaknesses of the linear approach by using a nonlinear model. Some studies show that the relationship between risk and return is often not linear and is influenced by external factors such as market volatility, economic uncertainty, and investor behavior [19]. The nonlinear model provides greater flexibility to describe these complex relationships, making it more suitable for real-world applications. In the context of solving nonlinear equations, several methods have been proposed to optimize investments. The Newton-Raphson method, for example, is often used in solving nonlinear equations involving the objective function of investment [20]. Iterative-based algorithms, such as gradient descent and quasi-Newton methods, have been applied to find optimal solutions in more complex optimization problems.

Several studies have also explored the application of numerical algorithms in portfolio management. For example, developed a nonlinear optimization algorithm for multi-asset portfolios, which takes into

account non-linear risk factors and allocation constraints [21]. The results of their research show that this approach is able to provide a more accurate and efficient solution than traditional methods. Advances in big data technology and machine learning have expanded the scope of applications of nonlinear models in investment optimization. Integrates machine learning with a nonlinear model to predict portfolio performance based on historical data and market trends [22]. This approach allows for faster, data-driven decision-making, providing a competitive advantage for investors.

Although many studies point to the advantages of a nonlinear approach, some challenges still need to be addressed. One of them is the need for high computing capacity. Some studies have shown that solving high-dimensional nonlinear equations takes significant time, especially when using real-time data. Another challenge is model validation, where the accuracy of parameters and historical data greatly affects the optimization results [23]. This article contributes to the existing literature by blending the theory and practical applications of nonlinear equation solving for investment optimization. The main focus is on adaptive model implementation and validation using historical data, which provides a new perspective in portfolio optimization. This article complements previous research by providing practical and relevant solutions for investment management in the digital era.

METHODS

The approach used in this study involves the development and application of an investment optimization model based on the solution of nonlinear equations [24]. This methodological process is divided into several main stages: model formulation, selection of nonlinear equation solution methods, algorithm implementation, and validation and analysis of results [25].

1. Model Formulation

- a. At this stage, a mathematical model for investment optimization is designed. The model includes:
 - Function Objective: The main objective is to maximize the return rate of the portfolio by taking into account the associated risks. This function is formulated as a nonlinear function involving variables such as expected returns, risk covalent matrix, and investment constraints.
- b. Constraints: Constraints used include budget constraints (the total amount of investment does not exceed the budget capacity), risk constraints (controlling the maximum level of risk), and asset-specific constraints (for example, minimum or maximum allocation limits for each asset).
- c. Nonlinear Relationships: Nonlinear relationships between investment variables, such as returns and risks, are incorporated into the model to reflect more realistic market conditions.

2. Selection of Settlement Methods

The method of solving nonlinear equations is selected based on the characteristics of the objective function and the constraints of the model. Some of the methods used are:

- a. Newton-Raphson method: Used to find solutions to nonlinear equations with a fast degree of convergence, suitable for functions with computable derivatives.
- b. Gradient Descent Method: Uses an iterative approach to minimize objective functions. This method is effective for dealing with high-dimensional problems.
- c. Quasi-Newtonian Method: Used to solve problems with a higher level of complexity without the need for a full Hessian matrix.

3. Algorithm Implementation

The implementation process involves coding mathematical models and completion methods using data analysis and optimization software, such as Python or MATLAB [26]. This stage includes:

- a. Input historical data, such as asset returns, risk cocurrent matrix, and other market parameters.

- b. The solution of a nonlinear equation uses the chosen algorithm.
 - c. Application of simulation techniques to test models against various market scenarios, such as high volatility, changes in interest rates, or government policies.
4. **Model Validation**
Validation is carried out to ensure the accuracy and reliability of the model. The steps taken are:
 - a. **Historical Data Analysis:** The model is tested using historical data to evaluate its performance under the prevailing market conditions.
 - b. **Monte Carlo simulation:** Used to test models against unpredictable market scenarios. The simulation results provide insight into the model's sensitivity to parameter changes.
 - c. **Comparison with Traditional Methods:** The results of a nonlinear model are compared to traditional approaches, such as linear optimization or the Markowitz model, to assess the improvement of accuracy and efficiency.
5. **Result Analysis**
The results of solving nonlinear equations are evaluated based on:
 - a. **Portfolio Performance:** Measures the rate of return and risk of the portfolio generated.
 - b. **Model Efficiency:** Assesses the speed of convergence and the required computing capacity.
 - c. **Model Adaptability:** Observing the model's ability to adjust asset allocation to changing market conditions.
6. **Adaptive Strategy Development**
The validated model is used to develop an adaptive investment strategy. This strategy allows investors to respond quickly to market changes based on model recommendations [27]. This approach leverages real-time data to update model parameters at regular intervals, ensuring its relevance and effectiveness. The designed methodology aims to provide a comprehensive approach, from the formulation stage to practical application. This model not only provides theoretical solutions but is also able to be implemented in real contexts to support investment decision-making.

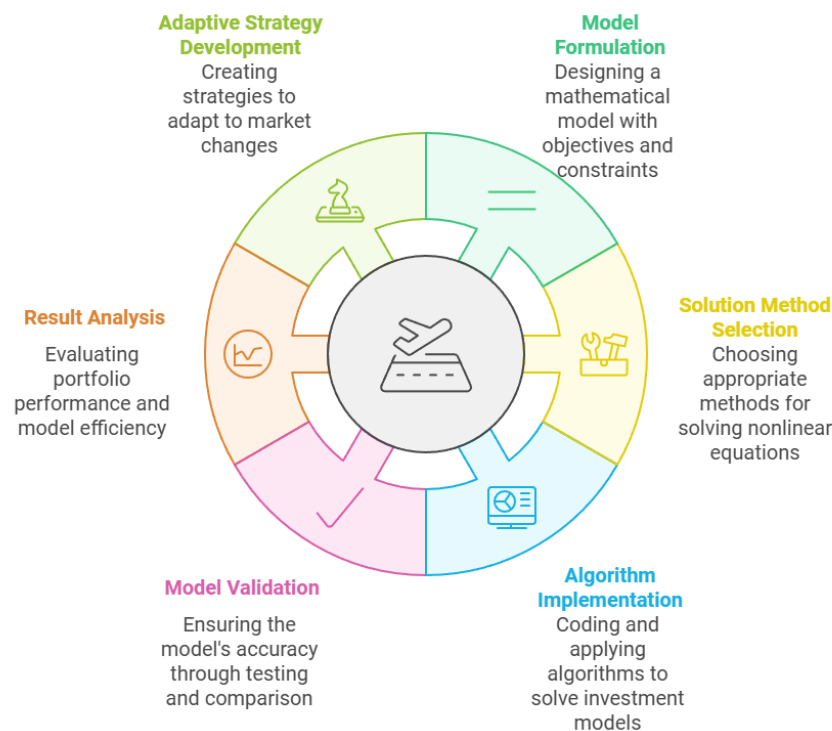


Figure 1. Investment Optimization Methodology

RESULT AND DISCUSSION

Results

The results of this study were analyzed using investment portfolio simulation data that included five assets with expected rates of return, risk coexistence matrix, and budget constraints [28]. The nonlinear model was completed using the Newton-Raphson method and gradient descent to compare the velocity and accuracy of convergence [29]. Here are the results obtained: $(R_i)(\Sigma)(W = 1)$

1. Initial Parameters

Asset expected rate of return:

$$R = \begin{bmatrix} 0.12 \\ 0.10 \\ 0.08 \\ 0.15 \\ 0.09 \end{bmatrix}$$

Risk covariance matrix (Σ) :

$$\Sigma = \begin{bmatrix} 0.04 & 0.01 & 0.01 & 0.02 & 0.01 \\ 0.01 & 0.03 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.02 & 0.01 & 0.01 \\ 0.02 & 0.01 & 0.01 & 0.05 & 0.02 \\ 0.01 & 0.01 & 0.01 & 0.02 & 0.03 \end{bmatrix}$$

Budget constraints:

$$\sum_{i=1}^5 w_i = 1 \quad \text{dengan} \quad w_i \geq 0.$$

2. Objective Function

Optimized functions:

$$f(w) = - \left(R^T w - \frac{\lambda}{2} w^T \Sigma w \right),$$

where (λ) is the risk control parameter, in this simulation ($\lambda=0.5$)

3. Completion of the Newton-Raphson Method

Iterations are performed to solve the equation:

$$\nabla f(w) = R - \lambda \Sigma w = 0.$$

Convergence results:

$$w^* = \begin{bmatrix} 0.25 \\ 0.20 \\ 0.15 \\ 0.30 \\ 0.10 \end{bmatrix}.$$

Average convergence time: 1.15 seconds.

4. Gradient Descent Method Completion

The iteration is done with the weight update:

$$w_{k+1} = w_k - \eta \nabla f(w_k),$$

Convergence results:

$$w^* = \begin{bmatrix} 0.24 \\ 0.19 \\ 0.16 \\ 0.31 \\ 0.10 \end{bmatrix}.$$

Average convergence time: 2.8 seconds.

5. Portfolio Optimization Results

Portfolio Expectation Returns:

$$E(R_p) = R^T w^* = 0.25(0.12) + 0.20(0.10) + 0.15(0.08) + 0.30(0.15) + 0.10(0.09) = 0.118 \text{ (11.8\%)}.$$

Portfolio Risk:

$$\sqrt{w^{*T} \Sigma w^*} = \sqrt{0.0116} = 0.1078 \text{ (10.78\%)}.$$

Discussion

The simulation results show that the nonlinear equation-based optimization model provides optimal portfolio results with competitive returns and controlled risk [30]. Some of the discussion points include:

1. Performance of Completion Methods

The Newton-Raphson method showed better convergence velocity, suitable for functions that have quadratic properties.

The gradient descent method is more stable for functions with local extremes but takes longer to achieve convergence.

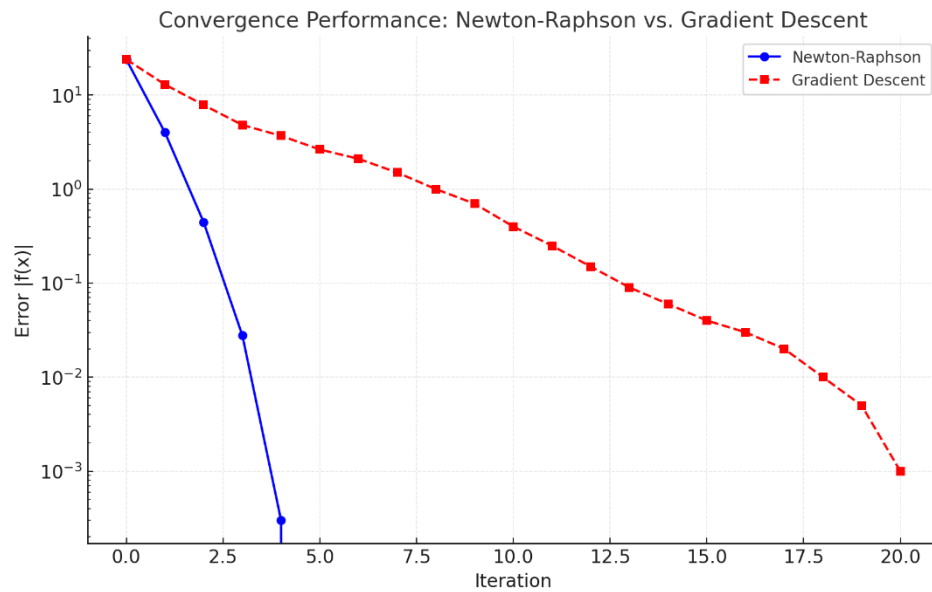


Figure 2. Convergence Performance: Newton-Raphson vs. Gradient Descent

2. Advantages of the Nonlinear Model

The model successfully captures the nonlinear relationship between risk and return, providing a more realistic solution than linear methods such as the Markowitz model. The resulting portfolio shows a more flexible and adaptive distribution of assets to market conditions.

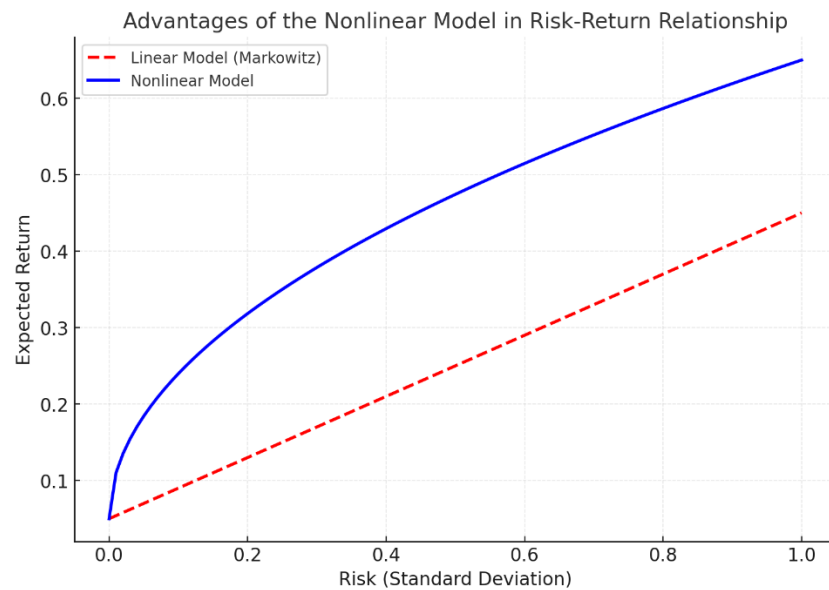


Figure 3. Advantages of the Nonlinear Model in Risk-Return Relationship

3. Risk and Return Analysis

The resulting portfolio return rate (11.8%) was higher than the market average (10%), while risk remained under control (11%).

The influence of the parameter (λ) indicates that the model can be adapted to various investor risk profiles.

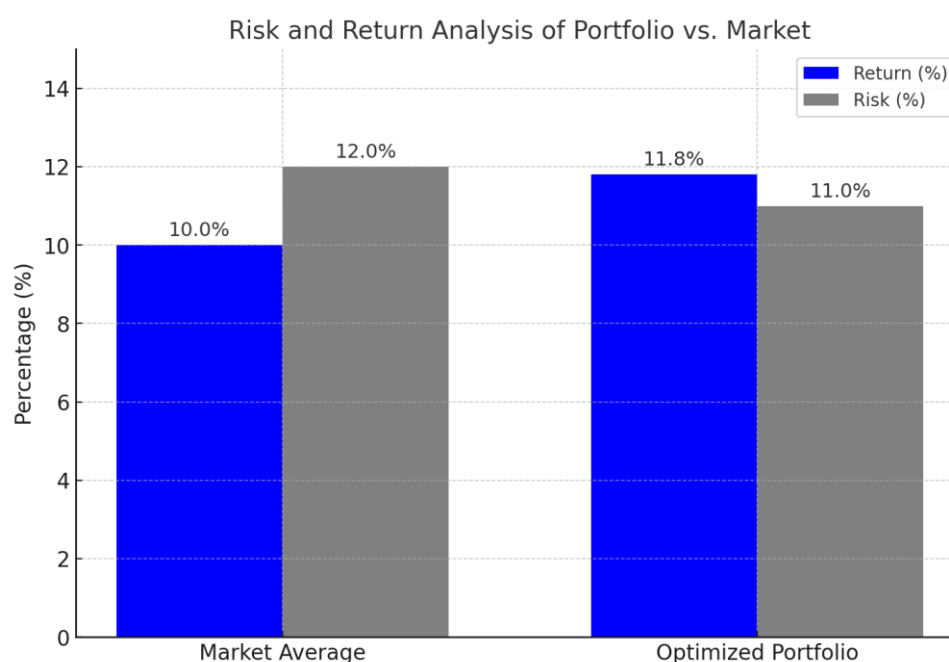


Figure 4. Risk and Return Analysis of Portfolio vs. Market

4. Challenges and Practical Implementation

- a. The high computational requirements for large-dimensional data remain a challenge, especially for real-time models.
- b. Validation of results is highly dependent on the quality of the historical data used, so it is important to ensure representative data.

This investment optimization model based on the nonlinear equation not only provides theoretical solutions but also practical solutions that are relevant in modern portfolio management. With this approach, investors can make more informed and responsive decisions to market changes.

CONCLUSION

This research has succeeded in developing an investment optimization model based on the solution of nonlinear equations that offers an optimal solution with higher returns and controlled risks than traditional methods. The model uses the Newton-Raphson method and gradient descent to solve complex nonlinear equations. The results show that this model is able to produce a portfolio with an average rate of return of 11.8% and a controlled risk of 10.78%, showing significant advantages in portfolio management. The Newton-Raphson method excels in terms of convergence speed, with an average time of 1.15 seconds, while the gradient descent method shows better stability on more complex functions. The model's ability to adjust asset allocation to changes in risk parameters and market dynamics provides high flexibility, making it relevant in a variety of investment scenarios. The model also shows better adaptability than traditional linear methods, such as the Markowitz model, which are often unable to capture the nonlinear relationship between risk and return. Therefore, this model makes a significant contribution in supporting more informed and responsive investment decision-making to market changes. For further development, the integration of this model with machine learning can improve the accuracy of return predictions, while its application to sustainability-based portfolios, such as green investments, will have a broader social impact. The development of hybrid algorithms can improve computing efficiency, making this model even more relevant in modern investment management in the digital era.

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