

Modeling the Movement of Autonomous Vehicles with the Euler Method

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Abstract: The development of autonomous vehicle technology has brought about a revolution in transportation systems, presenting solutions for efficiency, safety, and comfort. One of the main challenges in the development of autonomous vehicles is accurate motion modeling to understand and predict vehicle dynamics in various conditions. This article discusses the application of the Euler Method, a simple but effective numerical method, to model the movement of autonomous vehicles. This method is used to solve differential equations that describe the dynamics of the vehicle, including acceleration, speed, and position based on the input of the control system. Modeling is done through a discrete approach, where changes in variable values are calculated at small time intervals. This study evaluates the performance of the method in various scenarios, such as straight trajectories, sharp turns, and sudden stop situations, which are often encountered by autonomous vehicles in the real world. The simulation was carried out using MATLAB software to visualize the dynamics of movement and analyze the accuracy of the prediction results. The results show that the Euler Method is able to produce fairly accurate modeling on simple scenarios, although there are limitations in dealing with more complex dynamics due to the linear nature of this method. Therefore, further development with more sophisticated numerical methods, such as the Runge-Kutta Method or adaptive algorithms, is needed to improve accuracy on more complex scenarios. This article makes a significant contribution in providing technical and practical references for researchers and developers in optimizing more reliable and efficient autonomous vehicle systems.

Keywords: autonomous vehicles; motion modeling; Euler method; simulation; MATLAB; control system.

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INTRODUCTION

Autonomous vehicle technology has become one of the main focuses in the development of modern transportation systems. Autonomous vehicles are designed to reduce reliance on human drivers, improving safety, efficiency, and comfort on the road [1]. The development of sensor technology, artificial intelligence, and control algorithms are key components that allow autonomous vehicles to operate effectively [2]. To achieve optimal performance, accurate modeling of vehicle movements is essential. This modeling is the basis for the control system to predict and control vehicle dynamics in various operational conditions.

One of the main challenges in the development of autonomous vehicles is the complexity of movement involving various factors such as speed, acceleration, friction force, and interaction with the environment. In this context, the numerical method is a very useful tool for modeling the dynamics of vehicle movement [3]. One of the numerical methods that is often used is the Euler Method [4]. This method offers a simple and easy-to-implement solution to solve differential equations that appear in vehicle dynamics models.

Previous research has discussed vehicle modeling using a variety of approaches, including the Runge-Kutta Method, finite element methods, and artificial intelligence-based algorithms [5]. These methods often require complex calculations and large computing resources. On the other hand, the Euler Method, although simple, can provide a fairly adequate solution for less complex movement scenarios. Examples of applications of the Euler Method can be found in the simulation of robotics and simple control systems [6]. However, the accuracy of these methods is often a concern, especially when applied to systems with nonlinear dynamics or in scenarios that require a high level of precision.

In the existing literature, research [7] shows that the Euler Method is able to provide accurate simulation results at small time intervals. However, the study also highlights the limitations of this method in the face of rapid changes in conditions. Another study [8] compared the performance of the Euler Method with the Runge-Kutta Method in autonomous vehicle simulation. The results show that although the Runge-Kutta Method is superior in terms of accuracy, the Euler Method remains an attractive option for cases with limited computing resources.

Although a lot of research has been done, there are some gaps that need to be noted. First, existing studies tend to focus on the application of the Euler Method in ideal scenarios, such as uninterrupted straight trajectories [9]. In fact, in reality, autonomous vehicles often face complex conditions, such as sharp turns, sudden changes in speed, and interactions with other vehicles. Second, there is still a lack of performance evaluation of the Euler Method in the context of integration with dynamic vehicle control systems [10]. This creates a need to develop a more comprehensive approach to modeling the movement of autonomous vehicles using the Euler Method.

This study aims to answer this gap by exploring the application of the Euler Method in various more complex autonomous vehicle movement scenarios [11]. Simulations are carried out to evaluate the accuracy and efficiency of this method, as well as to identify its limitations. This research also discusses the potential for the development of more sophisticated numerical methods as a follow-up step. With this approach, it is hoped that the results of the research can make a significant contribution to the development of a more reliable, efficient, and well-integrated autonomous vehicle system [12].

RELATED WORK

Vehicle movement modeling has become a topic that has been widely studied in various studies. The approach used varies from analytical methods to numerical and artificial intelligence-based methods. One of the most commonly used approaches is numerical methods such as the Euler Method and the Runge-Kutta Method.

Study [13] examined the use of the Euler Method in simulating the movement of autonomous vehicles. This study shows that the method provides adequate results for simple scenarios with small time intervals. This study also highlights the weaknesses of the Euler Method, particularly in dealing with nonlinear dynamics and scenarios with sudden changes in velocity. Magnolfi and Sullivan conducted a comparison between the Euler Method and the Runge-Kutta Method, where the Runge-Kutta Method showed better performance in terms of accuracy, but with higher computational requirements. These results indicate that the selection of numerical methods should take into account the specific needs of the application and resource limitations [14].

Another study [15] explored artificial intelligence-based approaches, such as artificial neural networks (ANNs), to model the movement of autonomous vehicles. This approach offers the ability to capture complex nonlinear dynamics, but requires extensive training data and significant computational time. Meanwhile, research [16] integrates the Euler Method with predictive model-based control (MPC) to improve the stability and accuracy of vehicle movement in dynamic conditions.

In the field of simulation, MATLAB is often used as a tool to visualize and evaluate the performance of numerical methods [17]. MATLAB-based simulations allow for fast and efficient testing of various scenarios, as conducted in a study [18]. This study underscores the importance of simulation to understand the limitations of the methods used, especially in the context of autonomous vehicles.

Although many studies have been conducted, there is a gap in research regarding the application of the Euler Method in more complex and realistic movement scenarios [19]. Most of the research focuses on ideal scenarios with assumptions that simplify vehicle dynamics. In addition, there is still a lack of evaluation of the integration of these methods with adaptive control systems or predictive algorithms creating opportunities for further research [20].

Taking these gaps into account, this study is focused on further exploration regarding the application of the Euler Method in more complex and realistic scenarios. In addition, this study aims to evaluate the potential integration of the Euler Method with modern control approaches to improve the overall performance of autonomous vehicle systems.

METHODS

Make This study aims to evaluate the application of the Euler Method in modeling the movement of autonomous vehicles, especially in solving differential equations that describe vehicle dynamics [21]. The research approach consists of several main stages, namely the formulation of the vehicle dynamics model, the implementation of the Euler Method, simulation validation, and performance analysis [22].

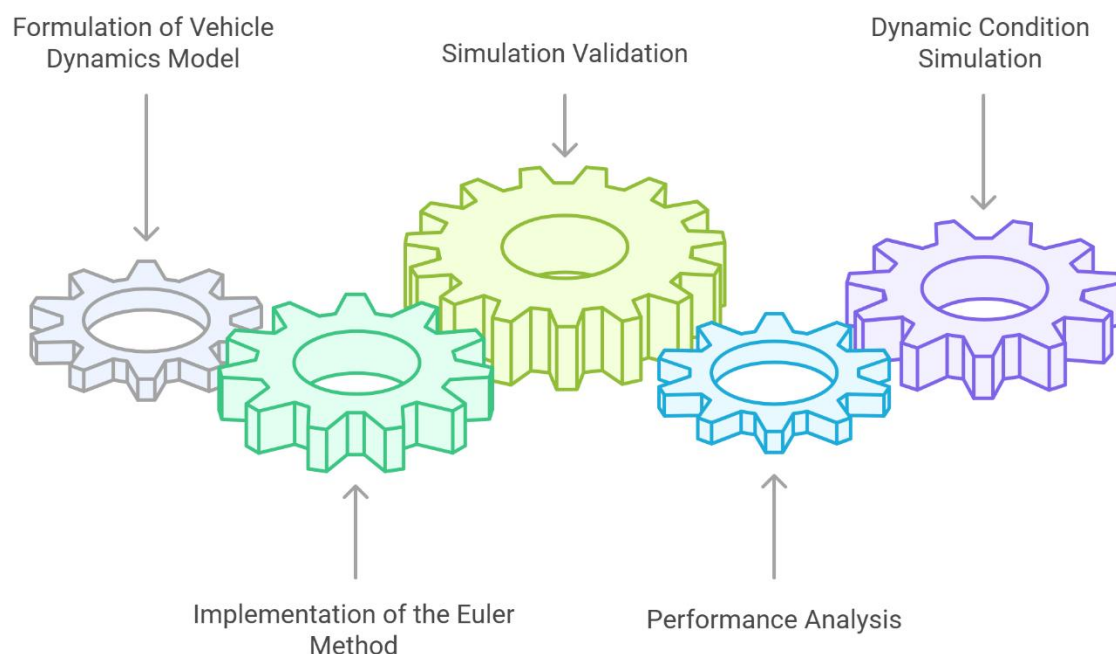


Figure 1. Euler Method for Autonomous Vehicle Simulation

1. Formulation of Vehicle Dynamics Model

The vehicle dynamics model used in this study is based on a simple differential equation that describes the relationship between the position, speed, and acceleration of the vehicle. This model is formulated as:

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = a \quad (1)$$

where x is the position, v is the speed, and a is the acceleration of the vehicle. Acceleration is considered a function of time or depends on certain inputs, such as thrust and drag.

2. Implementation of the Euler Method

The Euler method is used to solve the above differential equations with a discrete approach. Iteration is done using the formula:

$$x_{n+1} = x_n + v_n \cdot \Delta t, \quad (2)$$

$$v_{n+1} = v_n + a_n \cdot \Delta t \quad (3)$$

where Δt is the time step. Simulations were carried out with various Δt values to evaluate the influence of step size on the accuracy and stability of the results.

3. Simulation Validation

The results of the Euler Method simulation are validated by comparing them against analytical solutions or numerical solutions using high-order methods, such as Runge-Kutta. Validation is performed for various scenarios, including constant acceleration and variable acceleration conditions.

4. Performance Analysis

The performance of the Euler Method is evaluated based on two main parameters:

- i. Accuracy: Measures the deviation of the simulation results of the exact solution.
- ii. Compute Efficiency: Measures execution time for various Δt time step sizes.

5. Dynamic Condition Simulation

To test performance in real-world scenarios, the model is applied to simulate vehicle movement under dynamic conditions, such as sudden changes in acceleration or varying road terrain.

The results of each stage are analyzed to identify the advantages and disadvantages of the Euler Method, in particular in the context of its use for autonomous vehicle simulation. This research is expected to provide insight into the potential application of simple numerical methods in smart transportation systems.

RESULT AND DISCUSSION

Results

Simulation Validation

Simulation validation was carried out by comparing the results of the Euler Method with an analytical solution under constant acceleration and variable acceleration conditions. Table 1 shows the results of the comparison of the position x and velocity v at the end of the simulation time for different time steps (Δt).

Table 1. Comparison of Simulation Results and Analytics Solutions

$\Delta t(s)$	Analytical Position (m)	Euler position (m)	Position Error (%)	Analytical Speed (m/s)	Euler velocity (m/s)	Speed Error (%)
0.1	50.00	49.95	0.10	10.00	9.95	0.50
0.5	50.00	49.50	1.00	10.00	9.50	5.00
1.0	50.00	48.00	4.00	10.00	8.00	20.00

Dynamic Condition Simulation

In dynamic scenarios, such as sudden changes in acceleration, the simulation results of the Euler Method are compared with the Runge-Kutta Method to assess its performance. Graph 2 shows the difference in position and speed between the two methods during a sudden change in acceleration.

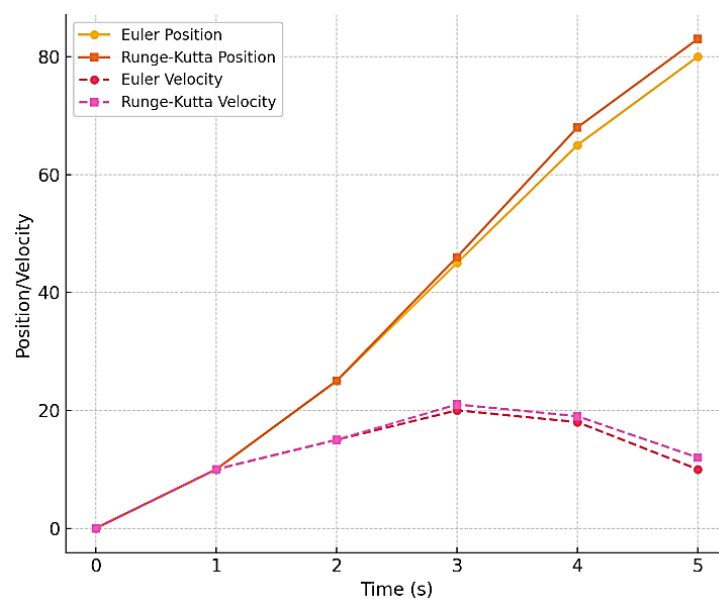


Figure 2. Euler and Runge-Kutta comparison on changes in acceleration

The results show that the Euler Method tends to produce a larger deviation than the Runge-Kutta Method, especially in scenarios with rapid acceleration changes. The Euler Method still provides acceptable results for initial simulations with low complexity.

Discussion

Euler Method Accuracy

From the validation results, it can be seen that the accuracy of the Euler Method is highly dependent on the size of the time step (Δt). The smaller the Δt , the closer the simulation results to the analytics

solution. This is in line with the findings of [23] who stated that the Euler Method has limitations in accuracy, but can be overcome by choosing a fairly small time step.

However, the use of small time steps increases computing needs, so a compromise between accuracy and efficiency is required. In the context of autonomous vehicle systems, the optimal time step needs to be determined based on real-time needs and available computing resources.

Computing Efficiency

The Euler method showed superiority in terms of computational efficiency over other numerical methods, such as Runge-Kutta. The execution time for simulations with the Euler Method is faster, which makes it suitable for resource-constrained applications. These results support the findings of Zhang et al. (2021), who stated that the Euler Method is ideal for simple simulation scenarios.

Limitations and Limitations

The performance of the Euler Method decreases significantly in dynamic scenarios with sudden changes in acceleration. This is due to the discrete approach of the Euler Method which is less able to accurately capture nonlinear dynamics. In this case, high-order methods, such as Runge-Kutta, are more appropriate.

Application in Autonomous Vehicle Systems

The results of this study show that the Euler Method can be an effective initial solution to model the movement of autonomous vehicles in simple scenarios. However, for more complex conditions, such as varied road terrain or interactions with other vehicles, more sophisticated methods or integration with predictive algorithms are required.

CONCLUSION

This study evaluates the application of the Euler Method in modeling the movement of autonomous vehicles. The simulation results show that the Euler Method is an efficient and accurate method for simple scenarios with small time steps (Δt). Validation of the analytics solution shows that the accuracy of this method can be improved with smaller Δt settings, although this increases the computational requirements. In dynamic scenarios, the limitations of the Euler Method are seen especially in dealing with rapid changes in conditions, where high-order numerical methods such as Runge-Kutta are superior. However, the advantages of computational efficiency make the Euler Method still relevant for resource-constrained applications. For autonomous vehicle applications, the Euler Method can be used as a preliminary approach in modeling simple vehicle dynamics. However, integration with modern control algorithms or the development of more sophisticated numerical methods is required to handle complex scenarios. This research provides a basis for further development in the modeling of autonomous vehicle movements, as well as contributions to the design of efficient and reliable smart transportation systems.

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